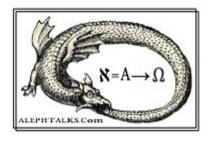
#### Mysticism: Where Science, Art and Religion Meet



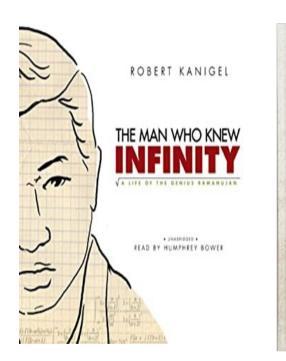


#### Subject Four Mysticism East and West Srinivasa Ramanujan ©AlephTalks 2022 12 January 2022



#### **Information Sources**

• Wikipedia



An Invitation to the ROGERS-RAMANUJAN IDENTITIES



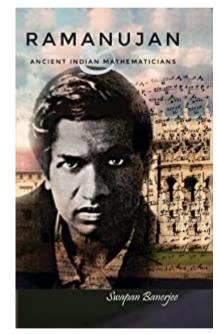
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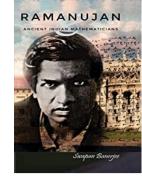
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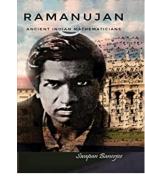
Ramanujan's Notebooks Part I



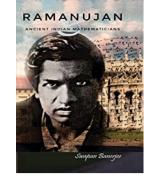




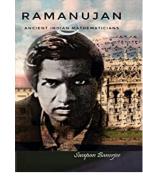
- Srinivasa Ramanujan FRS born Srinivasa Ramanujan Aiyangar, IPA 22 December 1887 – 26 April 1920) was an Indian mathematician who lived during the British Rule in India.
- Though he had almost no formal training in pure mathematics, he made substantial contributions to mathematical analysis, number theory, infinite series, and continued fractions, including solutions to mathematical problems then considered unsolvable.



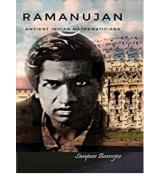
- Ramanujan initially developed his own mathematical research in isolation: according to Hans Eysenck: "He tried to interest the leading professional mathematicians in his work, but failed for the most part. What he had to show them was too novel, too unfamiliar, and additionally presented in unusual ways; they could not be bothered".
- Seeking mathematicians who could better understand his work, in 1913 he began a postal correspondence with the English mathematician G. H. Hardy at the University of Cambridge, England. Recognising Ramanujan's work as extraordinary, Hardy arranged for him to travel to Cambridge. In his notes, Hardy commented that Ramanujan had produced groundbreaking new theorems, including some that "defeated me completely; I had never seen anything in the least like them 01/19/2027 ore", and some recently proven but highly advanced results. 4



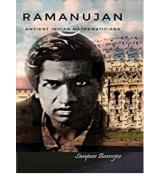
- During his short life, Ramanujan independently compiled nearly 3,900 results (mostly identities and equations). Many were completely novel; his original and highly unconventional results, such as the Ramanujan prime, the Ramanujan theta function, partition formulae and mock theta functions, have opened entire new areas of work and inspired a vast amount of further research.
- Of his thousands of results, all but a dozen or two have now been proven correct. The Ramanujan Journal, a scientific journal, was established to publish work in all areas of mathematics influenced by Ramanujan, and his notebooks containing summaries of his published and unpublished results —have been analysed and studied for decades since his death as a source of new mathematical ideas



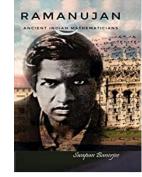
- As late as 2012, researchers continued to discover that mere comments in his writings about "simple properties" and "similar outputs" for certain findings were themselves profound and subtle number theory results that remained unsuspected until nearly a century after his death.
- He became one of the youngest Fellows of the Royal Society and only the second Indian member, and the first Indian to be elected a Fellow of Trinity College, Cambridge.
- Of his original letters, Hardy stated that a single look was enough to show they could have been written only by a mathematician of the highest calibre, comparing Ramanujan to mathematical geniuses such as Euler and Jacobi.



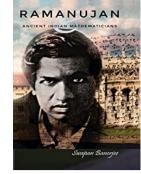
- In 1919, ill health—now believed to have been hepatic amoebiasis (a complication from episodes of dysentery many years previously)—compelled Ramanujan's return to India, where he died in 1920 at the age of 32. His last letters to Hardy, written in January 1920, show that he was still continuing to produce new mathematical ideas and theorems. His "lost notebook", containing discoveries from the last year of his life, caused great excitement among mathematicians when it was rediscovered in 1976.
- A deeply religious Hindu, Ramanujan credited his substantial mathematical capacities to divinity, and said the mathematical knowledge he displayed was revealed to him by his family goddess Namagiri Thayar. He once said, "An equation for me has no meaning unless it expresses a thought of God."



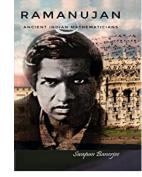
- In 1910, Ramanujan met deputy collector V. Ramaswamy Aiyer, who founded the Indian Mathematical Society. Wishing for a job at the revenue department where Aiyer worked, Ramanujan showed him his mathematics notebooks. As Aiyer later recalled:
  - I was struck by the extraordinary mathematical results contained in [the notebooks]. I had no mind to smother his genius by an appointment in the lowest rungs of the revenue department.
- Aiyer sent Ramanujan, with letters of introduction, to his mathematician friends in Madras. Some of them looked at his work and gave him letters of introduction to R. Ramachandra Rao, the district collector for Nellore and the secretary of the Indian Mathematical Society.



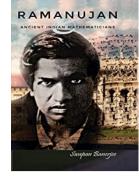
- Rao was impressed by Ramanujan's research but doubted that it was his own work. Ramanujan mentioned a correspondence he had with Professor Saldhana, a notable Bombay mathematician, in which Saldhana expressed a lack of understanding of his work but concluded that he was not a fraud.
- Ramanujan's friend C. V. Rajagopalachari tried to quell Rao's doubts about Ramanujan's academic integrity. Rao agreed to give him another chance, and listened as Ramanujan discussed elliptic integrals, hypergeometric series, and his theory of divergent series, which Rao said ultimately convinced him of Ramanujan's brilliance.
- When Rao asked him what he wanted, Ramanujan replied that he needed work and financial support. Rao consented and sent him to Madras. He continued his research with Rao's financial aid. With Aiyer's help, Ramanujan had his work published in the Journal of the Indian Mathematical Society.



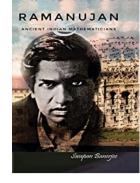
- One of the first problems he posed in the journal was to find the value of:
  - sqrt {1+2{\sqrt {1+3{\sqrt {1+... }}}}}.
- He waited for a solution to be offered in three issues, over six months, but failed to receive any.
- At the end, Ramanujan supplied the solution to the problem himself. On page 105 of his first notebook, he formulated an equation that could be used to solve the infinitely nested radicals problem.
  - x+n+a={\sqrt {ax+(n+a)^{2}+x{\sqrt {a(x+n)+(n+a)^{2}+(x+n){\sqrt {... }}}}}x+n+a={\sqrt {ax+(n+a)^{2}+x{\sqrt {a(x+n)+(n+a)^{2}+ (x+n){\sqrt {... }}}}}}
- Using this equation, the answer to the question posed in the Journal was simply 3, obtained by setting x = 2, n = 1, and a = 0.
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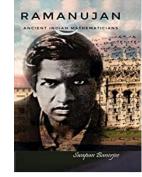
- Ramanujan wrote his first formal paper for the Journal on the properties of Bernoulli numbers. One property he discovered was that the denominators of the fractions of Bernoulli numbers are always divisible by six. He also devised a method of calculating Bn based on previous Bernoulli numbers. One of these methods follows:
  - It will be observed that if n is even but not equal to zero,
  - Bn is a fraction and the numerator of
  - Bn/n in its lowest terms is a prime number,
  - the denominator of Bn contains each of the factors 2 and 3 once and only once,
  - 2^n(2^n 1)\*Bn/n is an integer and 2(2^n 1)Bn consequently is an odd integer.
- In his 17-page paper "Some Properties of Bernoulli's Numbers" (1911), Ramanujan gave three proofs, two corollaries and three <sup>01/19/2022</sup> conjectures. His writing initially had many flaws.



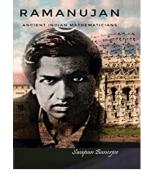
- As Journal editor M. T. Narayana Iyengar noted:
  - Mr. Ramanujan's methods were so terse and novel and his presentation so lacking in clearness and precision, that the ordinary [mathematical reader], unaccustomed to such intellectual gymnastics, could hardly follow him.
- Ramanujan later wrote another paper and also continued to provide problems in the Journal.
- In early 1912, he got a temporary job in the Madras Accountant General's office, with a monthly salary of 20 rupees. He lasted only a few weeks. Toward the end of that assignment, he applied for a position under the Chief Accountant of the Madras Port Trust



- In a letter dated 9 February 1912, Ramanujan wrote:
  - Sir,
  - I understand there is a clerkship vacant in your office, and I beg to apply for the same. I have passed the Matriculation Examination and studied up to the F.A. but was prevented from pursuing my studies further owing to several untoward circumstances. I have, however, been devoting all my time to Mathematics and developing the subject. I can say I am quite confident I can do justice to my work if I am appointed to the post. I therefore beg to request that you will be good enough to confer the appointment on me.



- Three weeks after he applied, on 1 March, Ramanujan learned that he had been accepted as a Class III, Grade IV accounting clerk, making 30 rupees per month.
- At his office Ramanujan easily and quickly completed the work he was given and spent his spare time doing mathematical research.
- Ramanujan's boss, Sir Francis Spring, and S. Narayana Iyer, a colleague who was also treasurer of the Indian Mathematical Society, encouraged Ramanujan in his mathematical pursuits.

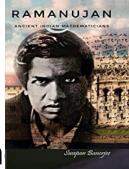


- In the spring of 1913, Narayana Iyer, Ramachandra Rao and E. W. Middlemast tried to present Ramanujan's work to British mathematicians. M. J. M. Hill of University College London commented that Ramanujan's papers were riddled with holes.
  - He said that although Ramanujan had "a taste for mathematics, and some ability", he lacked the necessary educational background and foundation to be accepted by mathematicians. Although Hill did not offer to take Ramanujan on as a student, he gave thorough and serious professional advice on his work. With the help of friends, Ramanujan drafted letters to leading mathematicians at Cambridge University.
- The first two professors, H. F. Baker and E. W. Hobson, returned Ramanujan's papers without comment

- On 16 January 1913, Ramanujan wrote to G. H. Hardy. Comfrom an unknown mathematician, the nine pages of mathematics made Hardy initially view Ramanujan's manuscripts as a possible fraud. Hardy recognised some of Ramanujan's formulae but others "seemed scarcely possible to believe".
- One result had already been determined by G. Bauer in 1859. The second was new to Hardy, and was derived from a class of functions called hypergeometric series, which had first been researched by Euler and Gauss. Hardy found these results "much more intriguing" than Gauss's work on integrals. After seeing Ramanujan's theorems on continued fractions on the last page of the manuscripts, Hardy said the theorems "defeated me completely; I had never seen anything in the least like them before", and that they "must be true, because, if they were not true, no one would have the imagination to invent <sup>01/18/2022</sup>. The second second

RAMANUJAN

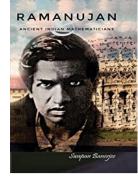
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- Hardy asked a colleague, J. E. Littlewood, to take a look at the papers. Littlewood was amazed by Ramanujan's genius. After discussing the papers with Littlewood, Hardy concluded that the letters were "certainly the most remarkable I have received" and that Ramanujan was "a mathematician of the highest quality, a man of altogether exceptional originality and power". One colleague, E. H. Neville, later remarked that "not one [theorem] could have been set in the most advanced mathematical examination in the world".
- On 8 February 1913 Hardy wrote Ramanujan a letter expressing interest in his work, adding that it was "essential that I should see proofs of some of your assertions". Before his letter arrived in Madras during the third week of February, Hardy contacted the Indian Office to plan for Ramanujan's trip to Cambridge. Secretary Arthur Davies of the Advisory Committee for Indian Students met with Ramanujan to discuss the overseas trip. In accordance with his Brahmin upbringing, Ramanujan refused to leave his country to "go to a foreign land". Meanwhile, he sent Hardy a letter packed with theorems, writing, "I have found a friend in you who views my labour sympathetically."



- To supplement Hardy's endorsement, Gilbert Walker, a form mathematical lecturer at Trinity College, Cambridge, looked at Ramanujan's work and expressed amazement, urging the young man to spend time at Cambridge. As a result of Walker's endorsement, B. Hanumantha Rao, a mathematics professor at an engineering college, invited Ramanujan's colleague Narayana Iyer to a meeting of the Board of Studies in Mathematics to discuss "what we can do for S. Ramanujan". The board agreed to grant Ramanujan a monthly research scholarship of 75 rupees for the next two years at the University of Madras.
- While he was engaged as a research student, Ramanujan continued to submit papers to the Journal of the Indian Mathematical Society.



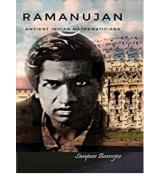
- In one instance Iyer submitted some of Ramanujan's theorem on summation of series to the journal, adding, "The following theorem is due to S. Ramanujan, the mathematics student of Madras University."
- Later in November, British Professor Edward B. Ross of Madras Christian College, whom Ramanujan had met a few years before, stormed into his class one day with his eyes glowing, asking his students, "Does Ramanujan know Polish?" The reason was that in one paper, Ramanujan had anticipated the work of a Polish mathematician whose paper had just arrived in the day's mail. In his quarterly papers Ramanujan drew up theorems to make definite integrals more easily solvable. Working off Giuliano Frullani's 1821 integral theorem, Ramanujan formulated generalisations that could be made to evaluate formerly unvielding integrals.



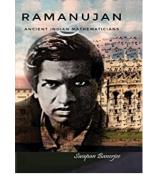
- Hardy's correspondence with Ramanujan soured after Ramanujan refused to come to England.
- Hardy enlisted a colleague lecturing in Madras, E. H. Neville, to mentor and bring Ramanujan to England. Neville asked Ramanujan why he would not go to Cambridge. Ramanujan apparently had now accepted the proposal; Neville said, "Ramanujan needed no converting" and "his parents' opposition had been withdrawn".
- Apparently Ramanujan's mother had a vivid dream in which the family goddess, the deity of Namagiri, commanded her "to stand no longer between her son and the fulfilment of his life's purpose". On 17 March 1914 Ramanujan traveled to England by ship leaving his wife to stay with his parents in India



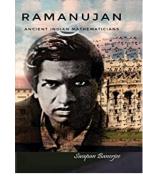
- Ramanujan departed from Madras aboard the S.S. Nevasa 17 March 1914. When he disembarked in London on 14 April, Neville was waiting for him with a car. Four days later, Neville took him to his house on Chesterton Road in Cambridge. Ramanujan immediately began his work with Littlewood and Hardy. After six weeks Ramanujan moved out of Neville's house and took up residence on Whewell's Court, a five-minute walk from Hardy's room.
- Hardy and Littlewood began to look at Ramanujan's notebooks. Hardy had already received 120 theorems from Ramanujan in the first two letters, but there were many more results and theorems in the notebooks. Hardy saw that some were wrong, others had already been discovered, and the rest were new breakthroughs. Ramanujan left a deep impression on Hardy and Littlewood. Littlewood commented, "I can believe that he's at least a Jacobi", while Hardy said he "can compare him only <sup>01/19/2022</sup> With Euler or Jacobi."



- Ramanujan spent nearly five years in Cambridge collaborating with Hardy and Littlewood, and published part of his findings there. Hardy and Ramanujan had highly contrasting personalities.
- Their collaboration was a clash of different cultures, beliefs, and working styles. In the previous few decades the foundations of mathematics had come into question and the need for mathematically rigorous proofs recognised. Hardy was an atheist and an apostle of proof and mathematical rigour, whereas Ramanujan was a deeply religious man who relied very strongly on his intuition and insights. Hardy tried his best to fill the gaps in Ramanujan's education and to mentor him in the need for formal proofs to support his results, without hindering his inspiration—a conflict that neither found easy.

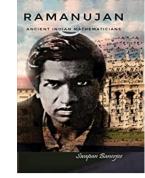


- Ramanujan was awarded a Bachelor of Arts by Research degree (the predecessor of the PhD degree) in March 1916 for his work on highly composite numbers, sections of the first part of which had been published the preceding year in the Proceedings of the London Mathematical Society. The paper was more than 50 pages long and proved various properties of such numbers. Hardy disliked this topic area but remarked that though it engaged with what he called the 'backwater of mathematics', in it Ramanujan displayed 'extraordinary mastery over the algebra of inequalities'.
- On 6 December 1917, Ramanujan was elected to the London Mathematical Society. On 2 May 1918, he was elected a Fellow of the Royal Society, the second Indian admitted, after Ardaseer Cursetjee in 1841. At age 31 Ramanujan was one of the youngest Fellows in the Royal Society's history. He was elected "for his investigation in elliptic functions and the Theory of Numbers." On 13 October 1918 he was the first Indian to be elected a Fellow of Trinity College, <sup>01/C2022</sup> Srinivasa Ramanujan

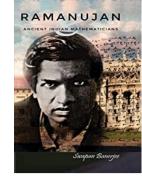


- Ramanujan was plagued by health problems throughout his life. His health worsened in England; possibly he was also less resilient due to the difficulty of keeping to the strict dietary requirements of his religion there and because of wartime rationing in 1914–18.
- He was diagnosed with tuberculosis and a severe vitamin deficiency, and confined to a sanatorium. In 1919 he returned to Kumbakonam, Madras Presidency, and in 1920 he died at the age of 32.
- After his death his brother Tirunarayanan compiled Ramanujan's remaining handwritten notes, consisting of formulae on singular moduli, hypergeometric series and continued fractions.

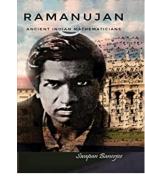
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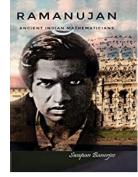
- Ramanujan's widow, Smt. Janaki Ammal, moved to Bombay; in 1931 she returned to Madras and settled in Triplicane, where she supported herself on a pension from Madras University and income from tailoring.
- In 1950 she adopted a son, W. Narayanan, who eventually became an officer of the State Bank of India and raised a family. In her later years she was granted a lifetime pension from Ramanujan's former employer, the Madras Port Trust, and pensions from, among others, the Indian National Science Academy and the state governments of Tamil Nadu, Andhra Pradesh and West Bengal.
- She continued to cherish Ramanujan's memory, and was active in efforts to increase his public recognition; prominent mathematicians, including George Andrews, Bruce C. Berndt and Béla Bollobás made it a point to visit her while in India. She died at her Triplicane residence in 1994.



- A 1994 analysis of Ramanujan's medical records and symptoms by Dr. D. A. B. Young concluded that his medical symptoms—including his past relapses, fevers, and hepatic conditions—were much closer to those resulting from hepatic amoebiasis, an illness then widespread in Madras, than tuberculosis.
- He had two episodes of dysentery before he left India. When not properly treated, amoebic dysentery can lie dormant for years and lead to hepatic amoebiasis, whose diagnosis was not then well established.
- At the time, if properly diagnosed, amoebiasis was a treatable and often curable disease; British soldiers who contracted it during the First World War were being successfully cured of amoebiasis around the time Ramanujan left England.



- Ramanujan has been described as a person of a somewhat shy and quiet disposition, a dignified man with pleasant manners. He lived a simple life at Cambridge. Ramanujan's first Indian biographers describe him as a rigorously orthodox Hindu. He credited his acumen to his family goddess, Namagiri Thayar (Goddess Mahalakshmi) of Namakkal. He looked to her for inspiration in his work and said he dreamed of blood drops that symbolised her consort, Narasimha. Later he had visions of scrolls of complex mathematical content unfolding before his eyes. He often said, "An equation for me has no meaning unless it expresses a thought of God."
- Hardy cites Ramanujan as remarking that all religions seemed equally true to him. Hardy further argued that Ramanujan's religious belief had been romanticised by Westerners and overstated—in reference to his belief, not practice—by Indian biographers. At the same time, he remarked on Ramanujan's strict vegetarianism



- Similarly, in an interview, Berndt said,
  - "Many people falsely promulgate mystical powers to Ramanujan's mathematical thinking. It is not true. He has meticulously recorded every result in his three notebooks," further speculating that Ramanujan worked out intermediate results on slate that he could not afford the paper to record more permanently."
- In mathematics there is a distinction between insight and formulating or working through a proof. Ramanujan proposed an abundance of formulae that could be investigated later in depth. G. H. Hardy said that Ramanujan's discoveries are unusually rich and that there is often more to them than initially meets the eye. As a byproduct of his work, new directions of research were opened up.